UPPSC-AE 2025

Uttar Pradesh
Public Service Commission

Combined State Engineering Services Examination

Assistant Engineer

Civil Engineering

Engineering Mechanics

Well Illustrated **Theory** *with*

Solved Examples and **Practice Questions**

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Engineering Mechanics

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CHAPTER 1

Units and Measurement

1.1 Introduction

Measurement of any physical quantity involves comparison with a certain basic, arbitrary chosen, internationally accepted reference standard called unit. The result of a measurement of a physical quantity is expressed by a number (or numerical measure) accompanied by a unit. Although the number of physical quantities appears to be very large, we need only a limited number of units for expressing all the physical quantities, since they are interrelated with one another. The units for the fundamental or base quantities are called fundamental or base units. The units of all other physical quantities can be expressed as a combinations of the base units. Such units obtained for the derived quantities are called derived units. A complete set of these units, both the base units and derived units, is known as the system of units.

1.2 Different Types of Measurement System

A complete set of units which is used to measure all kinds of fundamental and derived quantities is called a system of units. Here are the common system of units used in mechanics:

(a) **The FPS System :** It is the British Engineering system of units, which uses foot, pound and second as the three basic units for measuring length, mass and time respectively.

(b) **The C.G.S. System :** The C.G.S. system is the Gaussian system, which uses centimetre, gram and second as the three basic units for measuring length, mass and time respectively.

(c) **The M.K.S. System :** The M.K.S. system is based on metre, kilogram and second as the three basic units for measuring length, mass and time respectively.

Note: C.G.S., M.K.S. and SI are metric or decimal system of units. The F.P.S. system is not a metric system.

1.3 SI Unit System

The system of units used by scientists and engineers around the World is commonly called the metric system but since, 1960, it has been known officially as the international system, or SI.

The advantage of the SI system are:

(i) The system makes use of only one unit for one physical quantity, which means a rational system of units.

(ii) In this system, all the derived units can be easily obtained from the basic and supplementary units, which means it is a coherent system of units.

(iii) It is a metric system which means that multiplies and submultiples can be expressed as a power of 10.

In SI units, there are seven fundamentals units as given in table below:

SI UNITS			
Base Quantity	**Unit**	**Symbol**	**Definition**
Length	Metre	m	One metre is the length of the path travelled by light in vacuum in 1/299,792,458 of a second (1983)
Mass	Kilogram	kg	One kilogram is the mass of the prototype cylinder of platinum iridium alloy (whose height is equal to its diameter), preserved at the International Bureau of Weights and Measures at Serves, near Paris, France. (1901)
Time	Second	s	One second is the duration of 9,192,631,770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of Cesium-133 atom.(1967)
Electric current	Ampere	A	One ampere is the constant current ,which when maintained in each of the two straight parallel conductors of infinite length and negligible cross section ,held one metre apart in vacuum shall produce a force per unit length of 2×10^{-7} N/m between them. (1948)
Temperature	Kelvin	K	One kelvin is the fraction of (1/273.16) of the thermodynamic temperature of the triple point of the water. (1967)
Amount of substance	Mole	mol	One mole is the amount of substance which contains as many elementary entities as there are atoms in 0.012 kg of pure carbon-12. (1971)
Luminous intensity	Candela	cd	One candela is the luminous intensity in a given direction, of a source that emits monochromatic radiation of frequency 5.4 × 1014 Hz and that has a radiant intensity of 1/683 watt/steradian in that direction. (1979)

NOTE

1. π radian = 180°

$$1 \text{ radian} = \frac{180°}{\pi} = \frac{180° \times 7}{22} = 57.27°$$

Also, 1° (degree of arc) = 60′ (minute of arc) and 1′ (minute of arc)

= 60″ (seconds of arc)

2. **Relation between radian degree and minutes:**

$$1° = \frac{\pi}{180} \text{ rad} = 1.745 \times 10^{-2} \text{ rad}$$

$$\therefore \quad 1' = \frac{1°}{60} = \frac{1.745 \times 10^{-2}}{60} = 2.908 \times 10^{-4} \text{ rad} \simeq 2.91 \times 10^{-4} \text{ rad}$$

$$1'' = \frac{1°}{3600} = \frac{1.745 \times 10^{-2}}{3600} = 4.847 \times 10^{-6} \text{ rad} = 4.85 \times 10^{-6} \text{ rad}$$

1.4 Measurement of Basic Quantities

1.4.1 Measurement of Length

The concept of length in physics is related to the concept of distance in everyday life. Length is defined as the distance between any two points in space. The SI unit of length is metre.

Distances ranging from 10^{-5} m to 10^{2} m can be measured by direct methods. For example, a metre scale can be used to measure the distance from 10^{-3} m to 1 m, vernier calipers up to 10^{-4} m, a screw gauge up to 10^{-5} m and so on. The atomic and astronomical distances cannot be measured by any of the above mentioned direct methods. Hence, to measure the very small and the very large distances, indirect methods have to be devised and used. In Table below, a list of powers of 10 (both positive and negative powers) is given. Prefixes for each power are also mentioned. These prefixes are used along with units of length, and of mass.

Prefixes of Powers of Ten

Multiple	Prefix	Symbol	Sub multiple	Prefix	Symbol
10^{1}	deca	da	10^{-1}	deci	d
10^{2}	hecto	h	10^{-2}	centi	c
10^{3}	kilo	k	10^{-3}	milli	m
10^{6}	mega	M	10^{-6}	micro	μ
10^{9}	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p
10^{15}	peta	P	10^{-15}	femto	f
10^{18}	exa	E	10^{-18}	atoo	a
10^{21}	zetta	Z	10^{-21}	zepto	z
10^{24}	yotta	Y	10^{-24}	yocto	y

(i) Measurement of small distances:

Screw gauge and vernier caliper : (a) Screw gauge: The screw gauge is an instrument used for measuring accurately the dimensions of objects up to a maximum of about 50 mm. The principle of the instrument is the magnification of linear motion using the circular motion of a screw. The least count of the screw gauge is 0.01 mm (b) Vernier caliper: A vernier caliper is a versatile instrument for measuring the dimensions of an object namely diameter of a hole, or a depth of a hole. The least count of the vernier caliper is 0.1 mm.

(ii) Measurement of large distances : For measuring larger distances such as the height of a tree, distance of the Moon or a planet from the Earth, some special methods are adopted. Triangulation method, parallax method and radar method are used to determine very large distances.

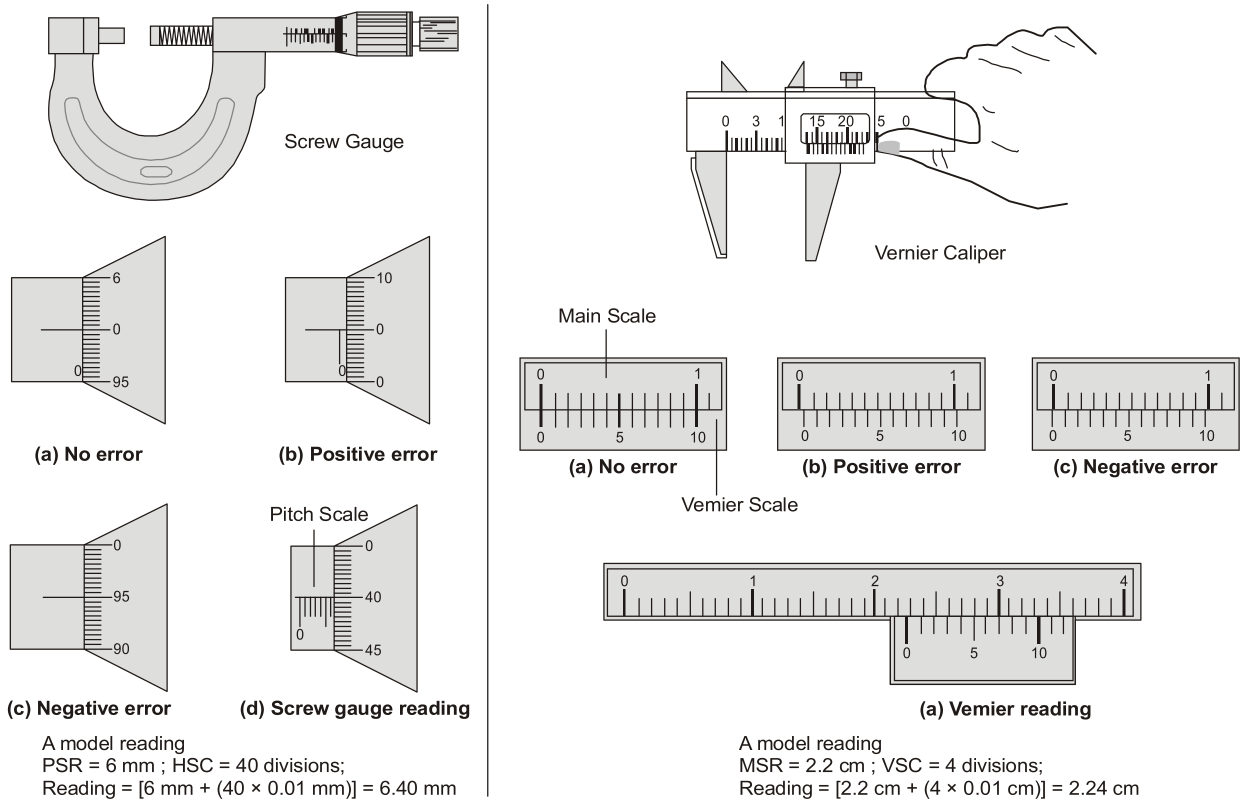

Figure : Screw Gauge and vernier caliper with error

Triangulation method for the height of an accessible object : Let $AB = h$ be the height of the tree or tower to be measured. Let C be the point of observation at distance x from B. Place a range finder at C and measure the angle of elevation, $\angle ACB = \theta$ as shown in Figure below. From right angled triangle

$$ABC, \tan\theta = \frac{AB}{BC} = \frac{h}{x} \quad \text{or} \quad \text{Height } h = x \tan\theta$$

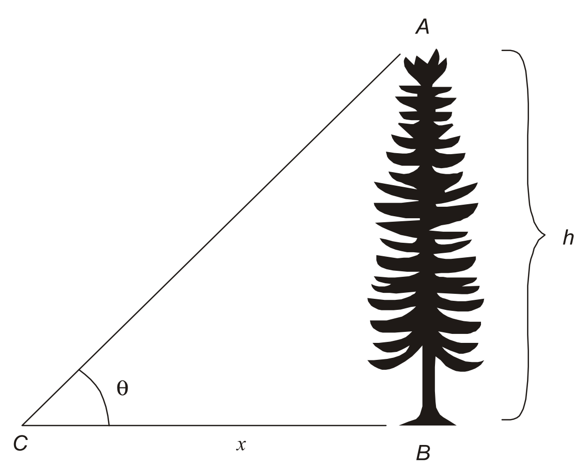

Figure : Triangulation method

Example - 1.1 From a point on the ground, the top of a tree is seen to have an angle of elevation 60°. The distance between the tree and a point of 50 m. Calculate the height of the tree?

Solution:

Angle $\theta = 60°$

The distance between the tree and a point $x = 50$ m

Height of the tree $(h) = ?$

For triangulation method $\tan\theta = \frac{h}{x}$

$$h = x\tan\theta = 50 \times \tan 60° = 50 \times 1.732 = 86.6 \text{ m}$$

1.4.2 Measurement of Mass

Mass is a property of matter. It does not depend on temperature, pressure and location of the body in space. Mass of a body is defined as the quantity of matter contained in a body. The SI unit of mass is kilogram (kg). The masses of objects which we shall study in this course vary over a wide range. These may vary from a tiny mass of electron (9.11×10^{-31} kg) to the huge mass of the known universe ($= 10^{55}$ kg). The order of masses of various objects is shown in Table below.

Object	Order of Mass (kg)	Object	Order of Mass (kg)
Electron	10^{-30}	Frog	10^{-1}
Proton or Neutron	10^{-27}	Human	10^{2}
Uranium atom	10^{-25}	Car	10^{3}
Red blood corpuscle	10^{-14}	Ship	10^{5}
A cell	10^{-10}	Moon	10^{23}
Dust Particle	10^{-9}	Earth	10^{25}
Raindrop	10^{-6}	Sun	10^{30}
Mosquito	10^{-5}	Milky way	10^{41}
Grape	10^{-3}	Observable Universe	10^{55}

Table : Range of Masses

Ordinarily, the mass of an object is determined in kilograms using a common balance like the one used in a grocery shop. For measuring larger masses like that of planets, stars etc., we make use of gravitational methods. For measurement of small masses of atomic/subatomic particles etc., we make use of a mass spectrograph. Some of the weighing balances commonly used are common balance, spring balance, electronic balance, etc.

1.4.3 Measurement of Time Intervals

A clock is used to measure the time interval. An atomic standard of time, is based on the periodic vibration produced in a Cesium atom. Some of the clocks developed later are electric oscillators, electronic oscillators, solar clock, quartz crystal clock, atomic clock, decay of elementary particles, radioactive dating etc.

1.5 Errors in Measurement

The uncertainty in a measurement is called an error. Random error, systematic error and gross error are the three possible errors.

1.5.1 Systematic errors

Systematic errors are reproducible inaccuracies that are consistently in the same direction. These occur often due to a problem that persists throughout the experiment. Systematic errors can be classified as follows:

(i) Instrumental errors: When an instrument is not calibrated properly at the time of manufacture, instrumental errors may arise. If a measurement is made with a meter scale whose end is worn out, the result obtained will have errors. These errors can be corrected by choosing the instrument carefully.

(ii) Imperfections in experimental technique or procedure: These errors arise due to the limitations in the experimental arrangement. As an example, while performing experiments with a calorimeter, if there is no proper insulation, there will be radiation losses. This results in errors and to overcome these, necessary correction has to be applied.

(iii) Personal errors : These errors are due to individuals performing the experiment, may be due to incorrect initial setting up of the experiment or carelessness of the individual making the observation due to improper precautions.

(iv) Errors due to external causes : The change in the external conditions during an experiment can cause error in measurement. For example, changes in temperature, humidity, or pressure during measurements may affect the result of the measurement.

(v) Least count error : Least count is the smallest value that can be measured by the measuring instrument, and the error due to this measurement is least count error. The instrument's resolution hence is the cause of this error. Least count error can be reduced by using a high precision instrument for the measurement.

1.5.2 Random errors

Random errors may arise due to random and unpredictable variations in experimental conditions like pressure, temperature, voltage supply etc. Errors may also be due to personal errors by the observer who performs the experiment. Random errors are sometimes called **"chance error".** When different readings are obtained by a person every time he repeats the experiment, personal error occurs. For example, consider the case of the thickness of a wire measured using a screw gauge. The readings taken may be different for different trials. In this case, a large number of measurements are made and then the arithmetic mean is taken.

If n number of trial readings are taken in an experiment, and the readings are a a_1, a_2, a_3, ... a_n. The arithmetic mean is

$$a_m = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n}$$

$$a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

Usually this arithmetic mean is taken as the best possible true value of the quantity.

1.5.3 Gross Error

The error caused due to the shear carelessness of an observer is called gross error. For example

(i) Reading an instrument without setting it properly.

(ii) Taking observations in a wrong manner without bothering about the sources of errors and the precautions.

(iii) Recording wrong observations.

(iv) Using wrong values of the observations in calculations.

These errors can be minimized only when an observer is careful and mentally alert.

1.6 Error Analysis

(i) Absolute Error: The magnitude of difference between the true value and the measured value of a quantity is called absolute error. If a_1, a_2, a_3, ... , a_n are the measured values of any quantity 'a' in an experiment performed n times, then the arithmetic means of these values is called the true value (a_m) of the quantity.

$$a_m = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} \text{ or } a_m = \frac{1}{n}\sum_{i=1}^{n} a_i$$

The absolute error in measured values is given by

$$|\Delta a_1| = |a_m - a_1|$$
$$|\Delta a_2| = |a_m - a_2|$$
$$\ldots\ldots\ldots\ldots$$
$$\ldots\ldots\ldots\ldots$$
$$|\Delta a_n| = |a_m - a_n|$$

(ii) Mean Absolute Error: The arithmetic mean of absolute errors in all the measurements is called the mean absolute error.

$$\Delta a_m = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots + |\Delta a_n|}{n} \text{ or } = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$$

(iii) Relative Error: The ratio of the mean absolute error to the mean value is called relative error. This is also called as fractional error (or) relative error. Thus

$$\text{Relative error} = \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{\Delta a_m}{a_m}$$

Relative error expresses how large the absolute error is compared to the total size of the object measured. For example, a driver's speedometer shows that his car is travelling at 60 km h^{-1} when it is actually moving at 62 km h^{-1}. Then absolute error of speedometer is 62-60 km h^{-1} = 2 km h^{-1}. Relative error of the measurement is 2 kmh^{-1}/60 km h^{-1} = 0.033.

(iv) Relative Error: The relative error expressed as a percentage is called percentage error.

$$\text{Percentage error} = \frac{\Delta a_m}{a_m} \times 100\%$$

A percentage error very close to zero means one is close to the targeted value, which is good and acceptable. It is always necessary to understand whether error is due to impression of equipment used or a mistake in the experimentation.

1.7 Dimensional Analysis

1.7.1 Dimension of Physical Quantities

All the derived physical quantities can be expressed in terms of some combination of the seven fundamental or base quantities. These base quantities are known as dimensions of the physical world, and are denoted with square bracket [ ]. The three dimensions in mechanics are [L] for length, [M] for mass and [T] for time. In electricity, [A] is the dimension of electric current. In thermodynamics, [K] is the dimension for the temperature. In optics [cd] or [Φ] is the dimension for luminous intensity. The dimension of amount of substance is [mol].

The dimensions of a physical quantity are the powers to which the units of base quantities are raised to represent a derived unit of that quantity.

For example,

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{[L]}{[T]} = [M^0LT^{-1}].$$

Hence the dimensions of velocity are 0 in mass, 1 in length –1 in time.

Dimensional formula and equation : Dimensional formula is an expression which shows how and which of the fundamental units are required to represent the unit of a physical quantity.

For example, $[M^0LT^{-2}]$ is the dimensional formula of acceleration.

When the dimensional formula of a physical quantity is expressed in the form of an equation, such an equation is known as the dimensional equation.

Example, acceleration = $[M^0LT^{-2}]$. The dimensional formula of various physical quantities are tabulated in Table.

Physical Quantity	Expression	Dimensional formula
Area	length × breadth	$[L^2]$
Volume	Area × height	$[L^3]$
Density	mass / volume	$[ML^{-3}]$
Velocity	displacement/time	$[LT^{-1}]$
Acceleration	velocity / time	$[LT^{-2}]$
Momentum	mass × velocity	$[MLT^{-1}]$
Force	mass × acceleration	$[MLT^{-2}]$
Work	force × distance	$[ML^2T^{-2}]$
Power	work / time	$[ML^2T^{-3}]$
Energy	Work	$[ML^2T^{-2}]$
Impulse	force × time	$[MLT^{-1}]$
Radius of gyration	Distance	$[L]$
Pressure (or) stress	force / area	$[ML^{-1}T^{-2}]$
Surface tension	force / length	$[MT^{-2}]$
Frequency	1 / time period	$[T^{-1}]$
Moment of Inertia	mass × (distance)^2	$[ML^2]$
Moment of force (or torque)	force × distance	$[ML^2T^{-2}]$
Angular velocity	angular displacement / time	$[T^{-1}]$
Angular acceleration	angular velocity / time	$[T^{-2}]$
Angular momentum	linear momentum × distance	$[ML^2T^{-1}]$
Co-efficient of Elasticity	stress/strain	$[ML^{-1}T^{-2}]$
Co-efficient of viscosity	(force × distance) / (area × velocity)	$[ML^{-1}T^{-1}]$
Surface energy	work / area	$[MT^{-2}]$
Heat capacity	heat energy / temperature	$[ML^2T^{-2}K^{-1}]$
Charge	current × time	$[AT]$
Magnetic induction	force / (current × length)	$[MT^{-2}A^{-1}]$
Force constant	force / displacement	$[MT^{-2}]$
Gravitational constant	[force × (distance)^2]/(mass)^2	$[M^{-1}L^3T^{-2}]$
Planck's constant	energy / frequency	$[ML^2T^{-1}]$
Faraday constant	avogadro constant × elementary charge	$[AT\ mol^{-1}]$
Boltzmann constant	energy / temperature	$[ML^2T^{-2}K^{-1}]$

Table : Dimensional Formula

1.7.2 Dimensional Quantities, Dimensionless Quantities, Principle of Homogeneity

On the basis of dimension, we can classify quantities into four categories.

(i) Dimensional variables : Physical quantities, which possess dimensions and have variable values are called dimensional variables. Examples are length, velocity, and acceleration etc.

(ii) Dimensionless variables : Physical quantities which have no dimensions, but have variable values are called dimensionless variables. Examples are specific gravity, strain, refractive index etc.

Student's Assignment

Q.1 Which of the following pairs of physical quantities have same dimension?
(a) force and power
(b) torque and energy
(c) torque and power
(d) force and torque

Q.2 The dimensional formula of Planck's constant h is
(a) $[ML^2T^{-1}]$ (b) $[ML^2T^{-3}]$
(c) $[MLT^{-1}]$ (d) $[ML^1T^{-3}]$

Q.3 The velocity of a particle v at an instant t is given by $v = at + bt^2$. The dimensions of b is
(a) $[L]$ (b) $[LT^{-1}]$
(c) $[LT^{-2}]$ (d) $[LT^{-3}]$

Q.4 Th e dimensional formula for gravitational constant G is
(a) $[ML^3T^{-2}]$ (b) $[M^{-1}L^3T^{-2}]$
(c) $[M^{-1}L^{-3}T^{-2}]$ (d) $[ML^{-3}T^2]$

Q.5 Th e density of a material in CGS system of units is 4 g cm^{-3}. In a system of units in which unit of length is 10 cm and unit of mass is 100 g, then the value of density of material will be
(a) 0.04 (b) 0.4
(c) 40 (d) 400

Q.6 If the force is proportional to square of velocity, then the dimension of proportionality constant is
(a) $[MLT^0]$ (b) $[MLT^{-1}]$
(c) $[MLT^{-2}T]$ (d) $[ML^{-1}T^0]$

Q.7 The dimension of $(\mu_0 \varepsilon_0)^{1/2}$ is
(a) length (b) time
(c) velocity (d) force

Q.8 Given that $F = (\alpha t^{-1} + \beta t^2)$ where F denotes force and t time; how is β described dimensionally?
(a) MLT^{-3} (b) MLT^{-2}
(c) LT^{-4} (d) MLT^{-4}

Q.9 Match **List-I** with **List-II** and select the correct answer using the codes given below the list:

List-I (Physical (Dimensional quantity)	List-II formula)
I. Specific gravity	A. $[M^0L^2T^{-1}]$
II. Coefficient of viscosity	B. $[M^0L^0T^0]$
III. Kinematic viscosity	C. $[ML^{-1}T^{-1}]$
IV. Stress	D. $[ML^{-1}T^{-2}]$

(a) I-B, II-C, III-D, IV-A
(b) I-D, II-A, III-B, IV-C
(c) I-A, II-D, III-C, IV-B
(d) I-B, II-C, III-A, IV-D

ANSWER KEY / STUDENT'S ASSIGNMENT

1. (b)	**2.** (a)	**3.** (d)	**4.** (b)	**5.** (c)
6. (d)	**7.** (c)	**8.** (d)	**9.** (c)	

HINTS & SOLUTIONS / STUDENT'S ASSIGNMENT

1. (b)
Dimension of torque = $[ML^2T^{-2}]$
Dimension of energy = $[ML^2T^{-2}]$

2. (a)
Dimension of planck; constant
$h = [ML^2T^{-1}]$

3. (d)
By the principle of homogeneity
Dimension of V = Dimension of bt^2
$[LT^{-1}] = b[T^2]$
∴ Dimension of $b = [LT^{-3}]$

5. (c)
The dimensional formula for density ρ is $[ML^{-3}]$